Introduction to Tidal Theory

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

1. INTRODUCTION

Tides: The periodic vertical movement of water on the Earth's Surface
(Admiralty Manual of Navigation)

Tides are very often neglected or taken for granted, "they are just the sea advancing and retreating once or twice a day." The Ancient Greeks and Romans weren't particularly concerned with the tides at all, since in the Mediterranean they are almost imperceptible. It was this ignorance of tides that led to the loss of Caesar's war galleys on the English shores, he failed to pull them up high enough to avoid the returning tide.

In the beginning tides were explained by all sorts of legends. One ascribed the tides to the breathing cycle of a giant whale. In the late 10th century, the Arabs had already begun to relate the timing of the tides to the cycles of the moon.

However a scientific explanation for the tidal phenomenon had to wait for Sir Isaac Newton and his universal theory of gravitation which was published in 1687. He described in his "*Principia Mathematica*" how the tides arose from the gravitational attraction of the moon and the sun on the earth. He also showed why there are two tides for each lunar transit, the reason why spring and neap tides occurred, why diurnal tides are largest when the moon was furthest from the plane of the equator and why the equinoxial tides are larger in general than those at the solstices. Thus the gravitational theory became established as the basis for all tidal science. Details of the tides at any given place are governed by the responses of the ocean to the gravitational forces.

South Africa has a relatively simple tidal system with semi-diurnal tides and a relatively small tidal range. Australia is however very complex from a tidal point of view. It has a full spectrum of tidal regimes, and a tidal range varying from 1m to over 10m. This is due to coastal topography and the shape of the ocean bottom which plays a major role in determining the tidal patterns around the country. In addition, we must also look at non-gravitational factors (eg seasonal, wind) that frequently have a large influence on the behaviour of the sea level. This can add additional complications to an already complicated situation.

Why do we need tidal predictions? Knowledge of the times and heights of tides and the speed and direction of tidal streams is important to a variety of people. These include:

a. <u>The Hydrographic Surveyor</u> in order to reduce soundings to a common datum. This is very important, as the Navy would not be able to defend our waters without accurate charts.

b. <u>The Navigator,</u> particularly in estuarine and coastal waters and the approaches to harbours.

c. Harbour and Coastal Engineers in the construction of harbour works, bridges, locks and dykes.

d. The Public to know when to go fishing, sailing, cross rivers on hiking trails etc.

2. TIDE RAISING FORCES

Firstly we need to look at Newton's laws of motion and gravity, however centripetal acceleration plays a major part in this section as well.

Newton's law of motion states that "the acceleration of a body equals the force acting on it per unit mass"

$$\text{Acceleration (a)} = \frac{\text{Force (F)}}{\text{Mass (m)}}$$

Newton's law of gravity states that "a body of mass M exerts a gravitational attraction on a unit of mass at a distance r of

$$Fg = \frac{GM}{r^2}$$

in which G is the universal gravitational constant (see definition list)

Centripetal Acceleration (A_c) is the acceleration of a body towards the center of curvature of the path along which it is moving and for a body with velocity along a path with radius of curvature (r).

$$(A_c) = \frac{v^2}{r}$$

We will now compare the gravitational attraction of the Sun on the Earth to that of the Moon on the Earth

Mass of the Sun = 27 million times that of the Mass of the Moon.
Distance of the Sun to Earth = 390 times the distance of the Moon to Earth

Thus $$\frac{Fg(sun)}{Fg(moon)} = \frac{27 \times 10^6}{(390)^2}$$

$$= 178 \text{ times that of the Moon.}$$

Introduction to Tidal Theory

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

So the gravitational attraction of the Sun is 178 times greater than that of the gravitational attraction of the Moon. But how can this be? We all know the moon is more effective in producing tides than the Sun. There is a simple explanation for this, and it is not that we have been lied to!

It is only the proportion of the gravitational force NOT balanced by centripetal acceleration (A_c) in the Earth's orbital motion that produces the tides. This unbalanced portion is proportional to the inverse cube of the distances rather than the inverse square of the distances from the Earth. However it is still proportional to the mass as in

$$Fg = \frac{GM}{.r^2}$$

And so from this we can see that the tide raising forces of the sun are approximately 178/390 = 0.46 times that of the Moon. Or the tide raising forces of the Sun are ½ that of the tide raising forces of the Moon

In general we talk about the Earth orbiting the Sun, but in reality the Earth and the Sun both rotate around a common center of mass which is less than 500km from the center of the Sun. Similarly the Moon and the Earth are orbiting about a common center of mass that is inside the Earth, approximately 1700km below the surface of the earth. It is the revolution of the Earth in this small orbit that is the counter part of the revolution about the sun.

We have already seen that the tide raising forces of the sun is only about ½ that of the moon. But we also need to look at the tide raising forces of the Moon in relation to that of the gravity of the Earth's surface. For this we can neglect all centrifugal forces due to axial rotation.

Upon comparison we see that the tidal forces of the moon is at most one ten-millionth of the earth's surface gravity. This may be thought of as negligable and thus insignificant, however these tiny forces act on every single particle of water throughout the depth of the ocean, accelerating them towards the sublunar (or subsolar) point on the near side of the Earth and towards the antipode on the farside. Thus the undulations set up in the deep oceans are quite gentle and only become prominent when their energy is compressed horizontally and vertically as they ride up into shallow and restricted coastal zones.

Revolution of the earth/moon system introduces a centripetal force. The horizontal component of the difference between the gravitational and centripetal force is what 'drives' the tides. These **horizontal tidal tractive forces** are very small and are inversely proportional to the cube of the distance between the earth and the moon but because they are not balanced forces they cause water movement. Their distribution on the earth's surface is shown in Fig 1

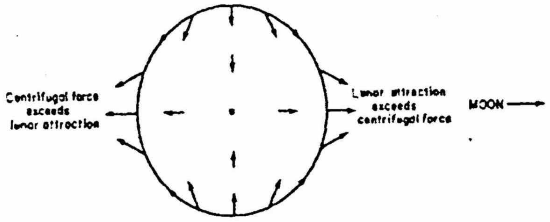

Fig 1 the distribution of the horizontal tidal tractive forces on the earth's surface

The pattern created by these tide producing forces produce bulges of water over the areas on the near and far sides to the moon where the forces are directed outward from the earth's surface. Depressions are noted between these areas where the forces are directed inwards. Fig 2 shows an imaginary earth covered with water, having no land masses, and the effect of the tide producing forces.

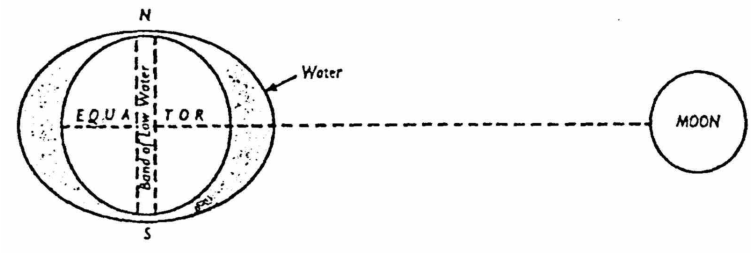

Fig 2 tidal bulges (semi-diurnal tide generation)

Now we introduce the rotation of the earth about it's polar axis; each point on the surface will pass through the whole pattern of forces in one day, with two passages under the bulges and two under the depressions. This is true for all points on the earth's surface, except near the poles. Hence the basis for the existence of the semi-diurnal tides on a diurnally rotating earth.

The Sun/Earth system sets up similarly to that of the Earth/Moon system, however the forces involved are much smaller as shown previously.

3. TIDAL PATTERNS.

Diurnal tides:

The origin of semi-diurnal tide has already been explained, but what about diurnal tides? We know that even semi-diurnal tides are often of unequal amplitude and in some places the tides are entirely diurnal. Thus we get diurnal inequalities. Diurnal tides are effected by the changes in solar and lunar declination. Fig 3 shows, in an exaggerated form, how the declination produces an asymmetry between the two high and two low water levels observed at the mid-latitude point **P** as it rotates under the two bulges. Both water levels are equal at the equator, but in high latitudes one high water disappears altogether.

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

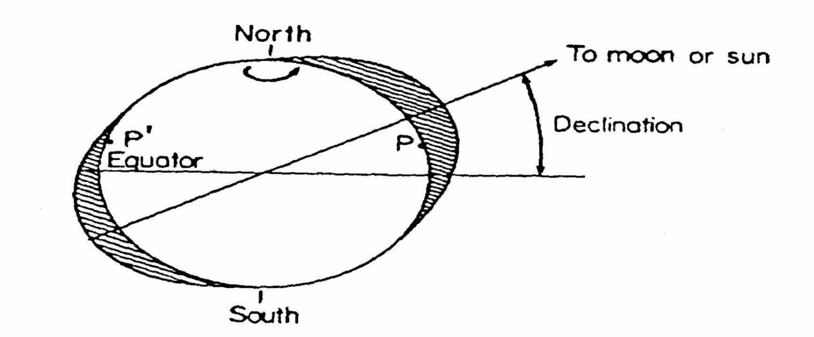

Fig 3 effects of declination (diurnal tide generation)

The maximum lunar monthly declination north and south of the equator varies from 18.3° to 28.6° over the 18.6 year nodal period. The solar declination varies seasonally from 23.5° in June to –23.5° in Dec. Thus the combined effect of the sun and the moon will increase the diurnal effect on the tides when their combined declinations are both large, either in the same or opposite sense. Thus the greatest diurnal effects occur when the moon is full, at solstices, ie in June and December in the Southern Hemisphere.

Spring and Neap tides:

It cannot be stressed enough that at no place on the Earth is the actual tide the same as the equilibrium tide at that place. Nevertheless, many of the characteristics of the two are similar except for magnitude and time. The sun's tide producing forces sometimes acts with or against those of the moon. At new and full moon, the sun and moon act together producing a large tide-raising force. During the spring tide, the high water's occur near local noon and local midnight and are higher than average due to the reinforcement of the two. The two low waters are also reinforced, but in the opposite sense, making them lower than average. The result is a larger than average range of semi-diurnal tide at spring tide. Actual big range tides spring tides occur a couple of days afterwards due to the inertia of the water mass. They occur at approximately fortnightly intervals.

With the moon at its first or last quarter, its tide-raising force is partially counteracted by that of the sun. Thus neap tides occur on the day that the sun's and moon's high waters most closely coincide with the others low waters. The result is a smaller than average range at neap tides. The resultant small range neap tides occur approximately a week after springs. Fig 4 illustrates the above.

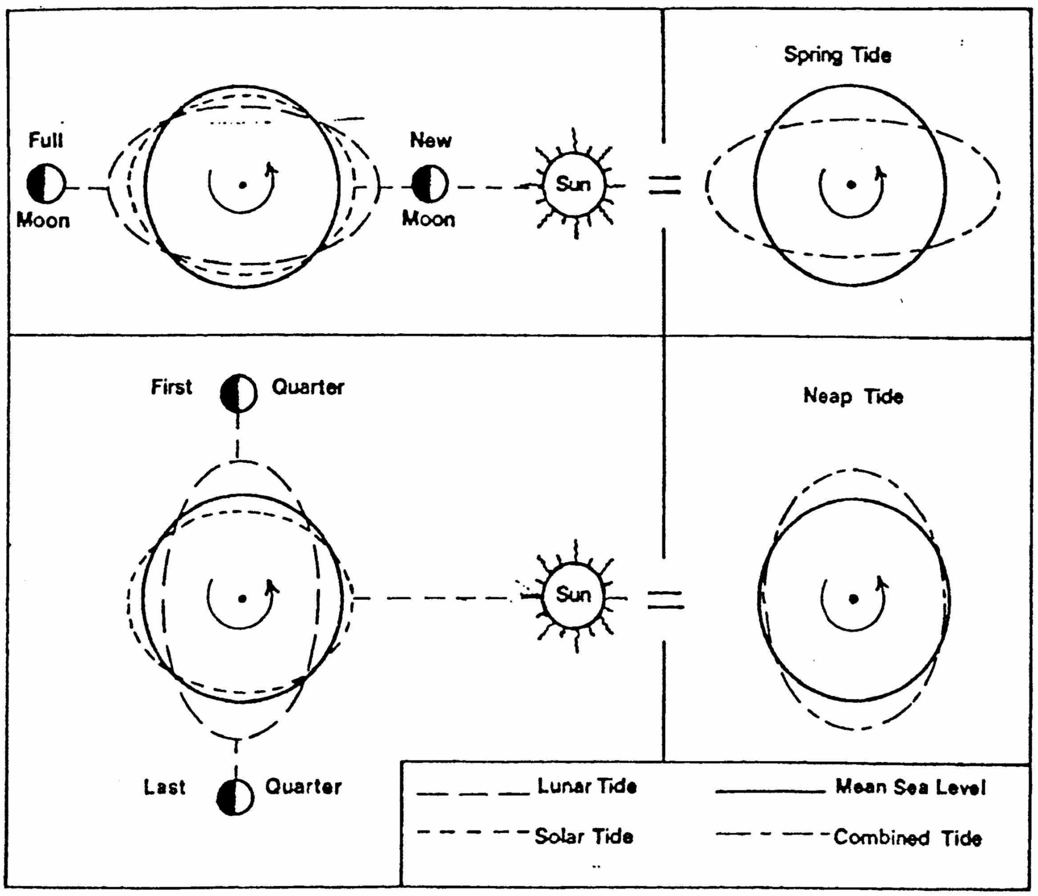

Fig 4 spring and neap tide generation.

NOTE: The range of the combined solar and lunar tides does not change suddenly at springs and neaps. It is however a sinusoidal modulation over the half-month period between successive springs or neaps.

4. REAL TIDE

Up until now we have been referring to tides on an imaginary earth that is totally covered in water and has no land masses/ continents. This is the basis for the **Equilibrium Tide** theory. The Equilibrium tide theory is defined as the elevation of the sea surface that would be in equilibrium with the tide forces if the earth were covered with water to such a depth that the response to these forces is instantaneous. In reality this has no resemblance to the real tides, and the rise and fall predicted by it are too small

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

compared to observed tides. This is however an important reference system for tidal analysis.

The **dynamic theory**/ real tide on the other hand, represents the tide as a wave "forced" by the tide-producing forces, and the rise and fall on the coast as a result of flow convergence or divergence. In theory, it allows calculations of tidal flows in the ocean, and the rise and fall on the shores. However, the real ocean basins have very complicated coastal and bottom topography and it is not possible to obtain exact solutions, except in the open sea.

There are several important factors that modify the movement of water in real tide situations:

a. The Sun/Moon: The moon's gravitational effect is greater than that of the suns due to it's closer proximity to the Earth, but acting sometimes in conjunction with the sun and sometimes in opposition it varies the amplitude and timing of the tides.

b. Geography: Land masses obviously impede and deflect movement of water on the Earth's surface.

c. Friction: Friction retards the movement of water particles across the Earth's surface – (the movement of tides across it is gradually slowing down the rotational speed of the Earth.)

d. Basin Oscillation: All bodies of water have natural periods of oscillation determined by their size and shape. All oceans are made up of a number of oscillating basins. The resultant oscillations at any one place affect the tidal movement or wave form depending upon the degree of resonance with the astronomic tidal curve.

e. Lunar and Terrestrial Orbits: The shape and plane of both the Earth's orbit around the Sun and the moon's orbit around the Earth are such that the distance between these bodies, their gravitational effect, varies continuously in cycles of months, years and even longer periods.

f. The Earth's Orbit: is in the form of an eccentric ellipse (eg or pear shaped). At perihelion the Earth is 91.3 million miles and at aphelion it is 94.5 million miles away from the sun respectively.

g. The Earth's Declination/ Tilt: 23° 27' off the vertical, hence the declination of the relative position of the sun and the moon as they appear to revolve around the Earth.

h. The Moon's Orbit: Also an eccentric ellipse with a varying apogee and perogee.

The Equilibrium theory explained above describes two bulges moving around the Earth from east to west at a steady rate. Their range would be 0.5m at the equator. This is not exactly what happens with the observed tides. The theoretical explanation of diurnal tides does not agree with the observations either. So why not?

The main reason for this complicated response to the tidal forcings is the fact that the land divides the world's waters into oceans, seas, gulfs etc. of different size, shape and depth. The only latitudes for the unimpeded circumpolar movement are around Antarctica and in the Arctic.

In addition, the water movements are affected by the rotation of the Earth. The Coriolis effect (which we will come to later) causes the water to take a curved path rather than a straight one and Kelvin waves produce different tidal ranges across channels. The best example of this is that of the English Channel where the French coast experiences a much larger tidal range than the British side

Natural Resonance

The various bodies of water have their individual natural periods of oscillation. This influences their response to the tide- raising force. The Pacific Ocean has, in general, a natural period of oscillation of about 25 hours, making it resonant to the diurnal components of the tide raising forces, so the tides tend to be diurnal there. The natural period of oscillation of the Atlantic is about 12.5 hours making it resonant to the semi-diurnal components and so the tides in that ocean are mainly semi-diurnal. Pacific tides are observed to have much more diurnal characteristics in general than Atlantic tides. There are also seas that have a natural period of oscillation that makes them unresponsive to either diurnal or semi-diurnal forces, these are known as non-tidal waters. Good examples of non-tidal waters are: Eastern Mediterranean, Baltic, Black and Caspian Seas

Coriolis and Friction.

Coriolis and friction are linked an understanding of both is important to understanding water movement.

Newton's law of motion applies only when all measurements are made with respect to an inertial coordinate system, that is, one that is neither accelerating nor rotating. However the Earth is rotating and so allowances need to be made for this. This is done by providing two "fictitious forces", the centrifugal force and the Coriolis force. The centrifugal force is conveniently combined with the Earth's gravitational force (G) in what we commonly refer to as "gravity" (g).

Introduction to Tidal Theory

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

The Coriolis force is rarely noticeable in laboratory-scale measurements, but it is very significant in large-scale geophysical motions such as winds, ocean currents and tides. In the upper wind driven layers of water bodies a balance is achieved between wind-stress, Coriolis force and pressure gradient field. The Coriolis force arises through relative motion on the rotating Earth and is proportional to the relative velocity and the sine of the latitude. It acts at right angles to the velocity, to the **right** in the **North**ern Hemisphere and to the **left** in the **South**ern Hemisphere. The Coriolis force without pressure gradients arising may balance the wind-stress. Thus a strong wind-stress leads to a strong Coriolis force.

Coriolis force acts in both the vertical and horizontal plane, but we will consider only the horizontal component. Imagine the earth to be covered with a frictionless film, the surface of which conforms to that of a level surface, ie is everywhere normal to the direction of gravity. As a body (or surface layer of water) moves to a higher latitude, the easterly velocity of the earth's surface decreases and so the easterly velocity of the body relative to the Earth increases. This is seen as an acceleration to the left in the Southern Hemisphere and an acceleration to the right in the Northern Hemisphere.

Now we need to ask the question, how does water move? Another major component of water movement is friction. But first we need to look at wind-stress.

The major current systems of the ocean are driven by the wind acting on the surface. The direct effect of the wind-stress is transmitted only to limited depth by viscosity and turbulence. The main surface current systems of the Atlantic and Indian oceans are in the form of large gyres that occupies most of the width of the ocean and are clockwise in the Northern hemisphere and anti-clockwise in the southern hemisphere. The Coriolis force is responsible for these circular patterns, deflecting both the winds and the currents driven by the winds.

The frictional forces that effect a particle in the ocean are:

a. **internal friction**, due mainly to eddy viscosity, which in shallow seas is often negligible compared with

b. **external friction**, due to stresses at the surface and at the sea-bed.

In 1905 V.W. Ekman, a Swedish mathematician and oceanographer, observed that icebergs in the Arctic ice pack were drifting at an angle to the direction of the wind. Ekman showed theoretically that the effect of wind blowing steadily over an ocean of infinite depth, extent and uniform eddy viscosity is to drive the surface layer at an angle ± 45° to the left of the wind direction in the Southern Hemisphere (to the right in the Northern Hemisphere) and to move the successive deeper layers of water more and more to the left until at a given depth the direction of the motion of water is opposite to that at the surface. Fig 5 In addition to the motion being directed more and more to the left (in the Southern Hemisphere), the speed of the motion decreases with depth, due to friction. This is known as the *Ekman Spiral*.

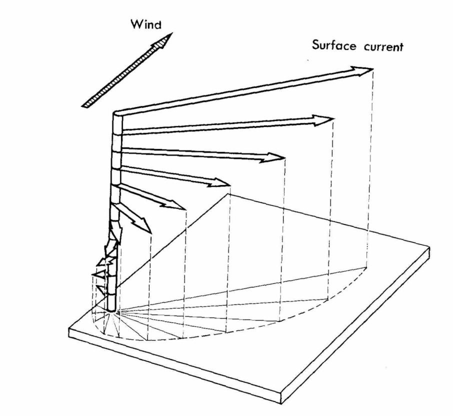

Fig 5 Schematic representation of wind driven current in deep water
(the Ekman Spiral) in the Northern Hemisphere

Usually, frictional influence is considered to cease at a depth of ± 100 meters where the direction of flow is opposite to that of the surface current. Here the flow has a speed approximately 1/23 as great as the speed on the surface. Ekman Spiral does not exist at the Equator as Coriolis is zero at the Equator.

Theoretically there is, in association with the rapid decay of velocity with depth in the Ekman Spiral, a net horizontal transport of water which is independent of any change of eddy viscosity with depth and is often observed. The net transport of water is thus at 90° to that of the wind direction (to the **right** in **north**ern hemisphere, to the **left** in the **south**ern hemisphere) This is known as Ekman Transport. Effect ascribed to Ekman transport are the upwelling (raising) or downwelling (sinking) that sometimes occurs when the wind is blowing with a large velocity parallel to the coastline. In the Southern Hemisphere a southerly wind blowing along the West Coast should drive surface layer water seawards. To replace this loss of water some upwelling will occur. Fig 6. If the deeper water along the coast is colder than that of the surface layer, the upwelling zone will be cooler than that some distance further at sea. Upwelling only occurs where the continental shelf is narrow.

Observations made to demonstrate the existence of the Ekman Spiral have not yet yielded clear confirmation of the theory in oceanic circumstances, but the statistical effects of Ekman transport on coastal climates and fishing grounds are well known. The West Coast of South Africa is a good example of this. All major commercial fishing is done on the West Coast of South Africa. Upwelling brings with it more nutrients and thus even though there is low diversity in fish life, the quantities are high due to the increases nutrient levels.

The oceans, in addition to being responsive to the effects of the wind, earth rotation, and gravity, are much influenced by the chemical and physical properties of the sea water itself.

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

Shallow Water Effects

Tides can be regarded as long waves, even in the open oceans as they have wavelengths much longer than the water depth. When a tide propagates into shallow water it's wave form is distorted from its sinusoidal form. In the shallows of the continental shelves, as the wave form's amplitude becomes a significant proportion of the water depth, their crests build up and the tidal range increases. At the same time however, the bottom friction removes more and more of the tidal energy, retards the trough and reduces the range. Irregular coastal topography further distorts the picture and exact mathematical solutions are rarely possible in these conditions.

Tides in the open ocean are usually of much smaller amplitude than those along the coast. As mentioned earlier, this is partly due to amplification by reflection and resonance. It is however, more generally the result of shoaling: as the wave propagates into shallower water, its wave speed decreases and the energy contained between crests is compressed both into a smaller depth and a shorter wavelength. The tide height and the tidal stream strength must increase accordingly. If, in addition, the tide propagates into an inlet whose width further diminishes toward the head, the wave energy if further compressed laterally. This effect, called *funneling*, also causes the tide height to increase.

Tidal bore is an extreme case of shallow water effects. It occurs usually at low water springs when a tide with a large range is funneled into a river or estuary with a steeply shelving bottom. Sometimes the front of the rising tide propagates up a river as a bore, a churning and tumbling wall of water advancing up a river not unlike a breaking surf riding up a beach. Creation of a bore requires a large rise of tide at the mouth of a river, some sandbars, or other restrictions at the entrance to impede the initial advance of the tide, and a shallow and gentle sloping river bed. Simply stated, the water cannot spread uniformly over the vast shallow interior area fast enough to match the rapid rise at the entrance. Friction at the base of the advancing front, plus resistance from the last ebb flow still leaving the river, causes the top of the advancing front to tumble forward, sometimes giving the bore the appearance of a travelling waterfall.

Amphidromic (Nodal) points. And rotating waves.

The word amphidromic is from the Greek for “a round race course”, and describes a system in which wave crests propagate like the spokes of a wheel around a central *amphidromic point*, with the wave amplitude increasing outward from zero at the centre.

In the Southern Hemisphere the earth's rotation can convert a simple standing wave in a basin into an amphidromic system (or amphidrome), in which the crest travels clockwise, tracing out an ellipse. The amplitude of the wave and of the particle velocity across the basin depends on the geometry and size of the basin and the length of the period of oscillation. The origin of and nature of amphidromes in the open ocean are less simple than those described above.

Channels and smaller seas receive tides by propagation from the amphidromes. These propagated tides are usually amplified and lead to tidal currents which we will discuss at a later stage.

Tidal streams are most noticeable in the straits and shallow waters. They are usually disregarded in the open ocean but they still exist there, although usually very weak. These streams usually never drop below zero, the vector of the constituent rotates around a centre called an amphidromic point, producing a rotary current.

The streams have a natural tendency to rotate in response to the tide raising forces. Additionally, the Coriolis effect deflects the movement on the rotating earth, thus amplifying the streams' natural tendency to rotate. The amphidromic centres, around which the streams rotate, move about as the sun and moon change their declination and distance

Storm Surges

The term wind set-up usually refers to the slope of the water surface in the direction of the wind stress. To achieve this steady state, the wind must be blowing for an extended period of time and the final effect is proportional to the fetch, and to the square of the wind speed. In addition, the wind-driven currents and the Coriolis effect cn create a water surface slope, but it is an indirect effect of the wind and generally not considered as wind set-up.

The combination of wind set-up and the inverted barometer effect associated with the storms can create a pronounced increase in sea level. This is often called a storm surge. An additional process in the form of a long surface wave travelling with the storm depression can further exaggerate this level increase. A negative surge, as the name implies, is the opposite effect, usually associated with high pressure systems and offshore winds, and can create unusually shallow water.

Tsunamis

The sudden movement of the sea floor upward or downward during a submarine earthquake can generate very large sea waves, incorrectly called “tidal waves.” Because the ocean tides have nothing to do with generating these huge waves, the Japanese term *tsunami* is preferred by scientists. Tsunamis are also called seismic sea waves. They generally are caused by great earthquakes (magnitude 8+) that disturb the sea floor, but they also result from submarine landslides or volcanic explosions. When a large section of the sea floor suddenly rises or falls during a quake, all water over the moving area is lifted or dropped for an instant. As the water returns to sea level, it sets up long, low waves that spread very rapidly over the ocean. Fig 8

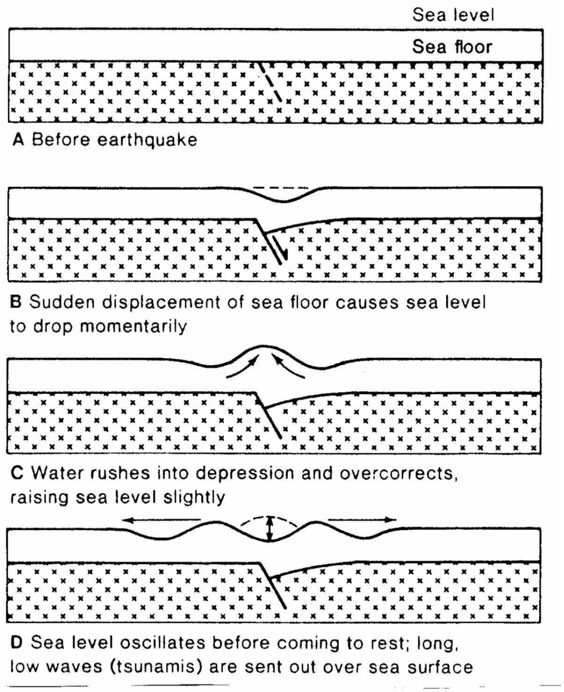

Fig 6 Generation of tsunamis by a submarine earthquake
(rock and water displacements are not drawn to scale)

Tsunamis are unlike ordinary water waves on the sea surface. A large wind generated wave may have a wavelength of 400m and be moving in deep water at a speed of 90 km/h. The wave height when it breaks on shore may only be 0.6 – 3 m, but near shore a tsunamis may peak up to heights of 15 – 30 m. This great increase in wave height near shore is caused by bottom topography; only a few localities have the combination of a gentle sloping offshore shelf and funnel shaped bay that force tsunamis to awesome heights (the record height was 85m in 1971 in the Ryukyu islands south of Japan.)

Although the speed of the wave slows dramatically as it moves through shallow water, a tsunami can still hit some shores as a very large, very fast wave. Because of its extremely long wavelength, a tsunami does not withdraw quickly as normal waves do. The water keeps on rising for five to ten minutes, causing great flooding before the wave withdraws. A tsunami’s long duration and great height can bring widespread destruction to the entire shore zone.

We cannot predict any of these non-tidal factors affecting sea level, and so the extreme events produce values far outside the predicted heights.

Introduction to Tidal Theory

Ruth Farre (BSc. Cert. Nat. Sci.)
South African Navy Hydrographic Office, Private Bag X1, Tokai, 7966

Reference books used

1. Bowditch, N (LL. D.): *American Practical Navigator.* Defense Mapping Agency Hydrographic Centre. 1977

2. Von Arx, W.S: *Introduction to Physical Oceanography.* Addison-Wesley Publishing Company, 1967

3. Emiliani, C: *Dictionary of the Physical Sciences: Terms, Formulas, Data.* Oxford University Press 1987

4. Hatfield, HR (Cdr RN): *Admiralty Manual of Hydrographic Surveying: Volume 2 Chapter 2: Tides and Tidal Streams*. The Hydrographer of the Navy, Taunton Somerset. 1969

5. Forrester. W.D. :*Canadian Tidal Manual*. Canadian Hydrographic Services, Ottawa 1983

6. University of Cape Town Ocean and Atmospheric Sciences course notes 1997-2001

7. Plummer/M^{c}Greary: *Physical Oceanogreaphy 7th Edition,* Wm. C. Brown Publishers 1996

8. W.G. van Dorn: *Oceanography and Seamanship 2nd Edition*, Cornell Maritime Press 1993